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# Study of temperature inversion symmetry for the twisted Wess–Zumino model

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**Abstract**
The temperature inversion symmetry, for a non-interacting supersymmetric ensemble, at finite volume, is studied. It is found that the scaled free energy, $f(\xi)$, is antisymmetric under temperature inversion transformation, i.e. $f(\xi) = -\xi^d f\left(\frac{1}{\xi}\right)$. This occurs for antiperiodic bosons and periodic fermions in the compact dimension. In contrast, for periodic bosons and antiperiodic fermions, $f(\xi) = \xi^d f\left(\frac{1}{\xi}\right)$.

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## 1. Introduction

One of the most interesting phenomena in quantum field theory is the Casimir effect. It expresses the quantum fluctuations of the vacuum of a quantum field. It originates from the 'confinement' of a field in finite volume. Many studies have been done since H Casimir's original work. The Casimir energy, usually calculated in these studies, is closely related to the boundary conditions of the fields under consideration. Boundary conditions influence the nature of the so-called Casimir force, which is generated from the vacuum energy.

By calculating the Casimir energy at finite temperature, one finds many interesting properties of fermionic and bosonic fields. In [2], the temperature inversion symmetry of the Casimir energy, for a spin-0 bosonic field, at finite temperature, was studied. The author actually examined some thermodynamic quantities for a bosonic field, at finite temperature and with a compact space dimension, in three spatial dimensions, i.e. a space having $S^1 \times R^2$ spatial topology. He found that the quantity,

$$f(\xi) = -\frac{L^{d-1}}{\Gamma\left(\frac{d}{2}\right)\xi^d \pi^{-\frac{d}{2}}} F, \tag{1}$$

i.e. the scaled free energy $f(\xi)$ of a spin-0 bosonic field ($\xi = LT$), with $F$,

$$F \sim T \sum_{n,m} \int \mathrm{d}k^2 \ln\left[k^2 + (2\pi n T)^2 + \left(\frac{2\pi m}{L}\right)^2\right], \tag{2}$$

is invariant under the transformation $L \to \frac{1}{T}$ ($L$ is the length of the compact dimension). As it can be easily seen from relation (2), he used periodic boundary conditions, corresponding to the compact dimension and periodic for the 'temperature dimension' (consistent with the KMS relations). In previous and later results [3–8], the symmetries, which the function (1) has, were studied and calculations for other fields, such as fermions and gauge fields, with various boundary conditions, were done. Studying thermodynamical quantities of fields in such topological spaces is of great importance (for example in microelectronics where characteristic distances become small [9]).

In this work, the free energy for an ensemble of massless, non-interacting bosons and fermions, with equal degrees of freedom, in spaces with $\frac{R^1}{Z_\infty} \times R^{d-2}$ spatial topologies at finite temperature, will be examined ($d$ is the total dimension of the spacetime before compactification, $Z_\infty$ the infinite cyclic group). Specifically, the temperature inversion symmetry shall be studied for this ensemble.

## 2. General set-up

One ensemble corresponding to massless, non-interacting bosons and fermions, with equal degrees of freedom, is the massless $N = 1, d = 4$, Wess–Zumino model. The massless, on the shell Wess–Zumino Lagrangian, is

$$\mathcal{L} = \partial_\mu \varphi^+ \partial^\mu \varphi + \mathrm{i}\overline{\Psi}\gamma^\mu \partial_\mu \Psi, \tag{3}$$

with $\Psi$ a Majorana spinor. Writing (3) in terms of two real fields $\phi_1, \phi_2, \varphi = \phi_1 + \mathrm{i}\phi_2$, we obtain

$$\mathcal{L} = \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \partial_\mu \varphi_2 \partial^\mu \varphi_2 + \mathrm{i}\overline{\Psi}\gamma^\mu \partial_\mu \Psi. \tag{4}$$

The total space-'time' is of the form $\mathcal{T} \otimes \frac{R^1}{Z_\infty} \times R^2$, $\mathcal{T}$ being the 'temperature time dimension'. Note that $Z_\infty$ must be a symmetry of the Lagrangian. The compactification of $R^1$ to $\frac{R^1}{Z_\infty}$ allows us to use generic boundary conditions for bosons and fermions in the compact dimension (of course there is some complication with supersymmetry and boundary conditions, for which, the reader is referred to the end of this section. The formulation adopted here is equivalent [1]). These are

$$\begin{aligned} \varphi_i(x_2, x_3, \tau, x_1) &= \mathrm{e}^{\mathrm{i}\pi n_1 \alpha}\, \varphi_i(x_2, x_3, \tau, x_1 + n_1 L) \\ \Psi(x_2, x_3, \tau, x_1) &= \mathrm{e}^{\mathrm{i}\pi n_1 \delta}\, \Psi(x_2, x_3, \tau, x_1 + n_1 L), \end{aligned} \tag{5}$$

with, $0 \leqslant \alpha, \delta \leqslant 1, i = 1, 2, n_1 = 1, 2, 3, \ldots,$ while the finite-temperature boundary conditions are (consistent with KMS relations)

$$\begin{aligned} \varphi_i(x_2, x_3, \tau, x_1) &= \varphi_i(x_2, x_3, \tau + \beta, x_1) \\ \Psi(x_2, x_3, \tau, x_1) &= -\Psi(x_2, x_3, \tau + \beta, x_1), \end{aligned} \tag{6}$$

with $\beta = \frac{1}{T}$. One can check that (4) is invariant for each representation of $Z_\infty$. The free energy at finite volume for the above ensemble is

$$F = F_{\text{bosonic}} + F_{\text{fermionic}}, \tag{7}$$

with $F_{\text{bosonic}}$, the total bosonic free energy (for $\phi_1, \phi_2$). Writing explicitly the free energies we obtain

$$\begin{aligned} F = T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^2}{(2\pi)^2} \ln\left[k^2 + (2\pi n T)^2 + \left(\frac{\pi(2m+\alpha)}{L}\right)^2\right] \\ - T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^2}{(2\pi)^2} \ln\left[k^2 + ((2n+1)\pi T)^2 + \left(\frac{\pi(2m+\delta)}{L}\right)^2\right]. \end{aligned} \tag{8}$$

For some choices of $\alpha$, and $\delta$, infrared singularities will occur, and of course an ultraviolet singularity. The UV will be regularized, using zeta regularization techniques [10] while, where necessary, infrared cutoffs will be used to deal with the infrared divergences [2].

*Case* $\alpha = 1, \delta = 0$. If one chooses $\alpha = 1, \delta = 0, n_1 = 1$, gets

$$F = T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^2}{(2\pi)^2} \ln\left[k^2 + (2\pi n T)^2 + \left(\frac{\pi(2m+1)}{L}\right)^2\right] - T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^2}{(2\pi)^2} \ln\left[k^2 + ((2n+1)\pi T)^2 + \left(\frac{2\pi m}{L}\right)^2\right]. \quad (9)$$

Relation (9) in $d$ dimensions (at the end $d = 4$) is written as

$$F = T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^{d-2}}{(2\pi)^{d-2}} \ln\left[k^2 + (2\pi n T)^2 + \left(\frac{\pi(2m+1)}{L}\right)^2\right] - T \sum_{n,m=-\infty}^{\infty} \int \frac{\mathrm{d}k^{d-2}}{(2\pi)^{d-2}} \ln\left[k^2 + ((2n+1)\pi T)^2 + \left(\frac{2\pi m}{L}\right)^2\right], \quad (10)$$

and upon using

$$\int \frac{\mathrm{d}k^d}{(2\pi)^d} \ln(k^2 + a^2) = -\frac{\Gamma\left(-\frac{d}{2}\right)}{(4\pi)^{\frac{d}{2}}} a^d, \quad (11)$$

(10) becomes

$$F = -T \frac{\Gamma\left(\frac{2-d}{2}\right)}{(4\pi)^{\frac{d-2}{2}}} \left( \sum_{n,m=-\infty}^{\infty} \left((2\pi n T)^2 + \left(\frac{\pi(2m+1)}{L}\right)^2\right)^{\frac{d-2}{2}} - \sum_{n,m=-\infty}^{\infty} \left(((2n+1)\pi T)^2 + \left(\frac{2\pi m}{L}\right)^2\right)^{\frac{d-2}{2}} \right).$$

By introducing the dimensionless parameter $\xi = LT$, we obtain

$$F = -T \frac{\Gamma\left(\frac{2-d}{2}\right)}{(4\pi)^{\frac{d-2}{2}}} \left(\frac{2\pi}{L}\right)^{d-2} \left( \sum_{n,m=-\infty}^{\infty} \left(\xi^2 n^2 + \left(m + \frac{1}{2}\right)^2\right)^{\frac{d-2}{2}} - \sum_{n,m=-\infty}^{\infty} \left(\xi^2 \left(n + \frac{1}{2}\right)^2 + m^2\right)^{\frac{d-2}{2}} \right). \quad (12)$$

In (12), the troublesome gamma function $\Gamma\left(\frac{2-d}{2}\right)$ appears, which will be cancelled later on. Now, with the aid of the two-dimensional form of the Epstein zeta function,

$$Z_2 \begin{vmatrix} g_1 & g_2 \\ h_1 & h_2 \end{vmatrix} (a, a_1, a_2) = \sum_{n,m=-\infty}^{\infty} (a_1(n+g_1)^2 + a_2(m+g_2)^2)^{-a} \times \exp[2\pi \mathrm{i}(n h_1 + m h_2)],$$

(12) becomes

$$F = -T \frac{\Gamma\left(\frac{2-d}{2}\right)}{(4\pi)^{\frac{d-2}{2}}} \left(\frac{2\pi}{L}\right)^{d-2} \left( Z_2 \begin{vmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{vmatrix} \left(\frac{2-d}{2}, \xi^2, 1\right) - Z_2 \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{vmatrix} \left(\frac{2-d}{2}, \xi^2, 1\right) \right). \quad (13)$$

To extend analytically the two-dimensional Epstein zeta, to values Re $a < 1$, we use the functional equation,

$$Z_2 \begin{vmatrix} g_1 & g_2 \\ h_1 & h_2 \end{vmatrix} (a, a_1, a_2) = (a_1 a_2)^{-\frac{1}{2}} \pi^{2a-1} \frac{\Gamma(1-a)}{\Gamma(a)} \times \exp[-2\pi i(g_1 h_1 + g_2 h_2)] \times Z_2 \begin{vmatrix} h_1 & h_2 \\ -g_1 & -g_2 \end{vmatrix} \left(1-a, \frac{1}{a_1}, \frac{1}{a_2}\right), \tag{14}$$

and (13) reads

$$F = -T \frac{\Gamma\left(\frac{2-d}{2}\right)}{(4\pi)^{\frac{d-2}{2}}} \left(\frac{2\pi}{L}\right)^{d-2} \left( (\xi^2)^{-\frac{1}{2}} \pi^{\frac{2-2d}{2}} \frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{2-d}{2}\right)} Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, \frac{1}{\xi^2}, 1\right) - (\xi^2)^{-\frac{1}{2}} \pi^{1-d} \frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{2-d}{2}\right)} Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, \frac{1}{\xi^2}, 1\right) \right). \tag{15}$$

Finally after some algebra we get

$$F = -\frac{\Gamma\left(\frac{d}{2}\right) \xi^d \pi^{-\frac{d}{2}}}{L^{d-1}} \left( Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) \right), \tag{16}$$

where the troublesome $\Gamma\left(\frac{2-d}{2}\right)$ (in four dimensions) gamma function has been cancelled. By introducing the function $f(\xi)$,

$$f(\xi) = -\frac{L^{d-1}}{\Gamma\left(\frac{d}{2}\right) \xi^d \pi^{-\frac{d}{2}}} F = \xi^d \left( Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) \right), \tag{17}$$

we can see that

$$f\left(\frac{1}{\xi}\right) = \frac{1}{\xi^d} \left( Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, 1, \frac{1}{\xi^2}\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, 1, \frac{1}{\xi^2}\right) \right), \tag{18}$$

or equivalently,

$$f\left(\frac{1}{\xi}\right) = Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, \xi^2, 1\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, \xi^2, 1\right). \tag{19}$$

From the last expression we easily obtain

$$Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, \xi^2, 1\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, \xi^2, 1\right) = \left( Z_2 \begin{vmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) \right). \tag{20}$$

Thus, combining (17), (19) and (20), we get

$$f(\xi) = -\xi^d f\left(\frac{1}{\xi}\right). \tag{21}$$

Relation (21) is a realization of an antisymmetry that the initial ensemble possesses, under the transformation $L \to \frac{1}{T}$, or equivalently, $\xi \to \frac{1}{\xi}$ (if we can say, this is a kind of anti-duality). The function $f(\xi)$ is related to the free energy of the system and, through this, to other thermodynamic quantities of the ensemble (it is also related to the effective potential). Let us check the physical implications of (21). The high temperature free energy is

$$F = -T^d \pi^{\frac{2-d}{2}} \Gamma\left(\frac{d}{2}\right) \left( Z_1 \begin{vmatrix} 0 \\ 0 \end{vmatrix} (d) - Z_1 \begin{vmatrix} 0 \\ -\frac{1}{2} \end{vmatrix} (d) \right), \tag{22}$$

which in $d = 4$ is

$$F = -T^4 \pi^{-2} \left( \frac{\pi^4}{45} + \frac{7\pi^4}{360} \right). \tag{23}$$

The zero-temperature Casimir energy for the same ensemble is

$$E_o = -L^{-d} \pi^{\frac{2-d}{2}} \Gamma\left(\frac{d}{2}\right) \left( Z_1 \begin{vmatrix} 0 \\ -\frac{1}{2} \end{vmatrix} (d) - Z_1 \begin{vmatrix} 0 \\ 0 \end{vmatrix} (d) \right) \tag{24}$$

which for $d = 4$ reads

$$E_o = \frac{1}{L^4} \pi^{-2} \left( \frac{\pi^4}{45} + \frac{7\pi^4}{360} \right). \tag{25}$$

Relations (23) and (25) reflect what (21) expresses that the high-temperature free energy of the boson fermion ensemble under consideration is equal to the minus zero temperature Casimir energy. This anti-duality found above was due to choosing antiperiodic boundary conditions for bosons ($\alpha = 1$), and periodic for fermions ($\delta = 0$ ), in the compact dimension. There is a mathematical reason for this choice. Isham [11] used these configurations, and called them twisted fields. These are classified by $H^1(S^1 \times R^2, Z_{\tilde{2}})$, the first Stieffel cohomology group which, in our case, is, $H^1(S^1 \times R^2, Z_{\tilde{2}}) = Z_2$ for the spatial section of $\mathcal{T} \otimes \frac{R^1}{Z_\infty} \times R^2$. Note that the temperature does not affect the topological properties of the space [12] and that $\frac{R^1}{Z_\infty} = S^1$. The allowed choices, dictated from $H^1(S^1 \times R^2, Z_{\tilde{2}})$, are periodic and antiperiodic bosons and fermions. Our choice was antiperiodic bosons and periodic fermions, which does not influence the supersymmetry transformations [11, 12]. In addition the Lagrangian is even under $H^1(S^1 \times R^2, Z_{\tilde{2}})$ for this choice. In the following section, antiperiodic fermions and periodic bosons shall be used (similarly this choice does not influence supersymmetry transformations).

*Case* $\alpha = 0, \delta = \frac{1}{2}$. Another case, which will be briefly mentioned, is the choice, $\alpha = 0$, $\delta = \frac{1}{2}, n_1 = 1$ in (8). Following the steps of the previous section, one easily obtains the free energy of the system:

$$F = -T \frac{\Gamma\left(\frac{2-d}{2}\right)}{(4\pi)^{\frac{d-2}{2}}} \left(\frac{2\pi}{L}\right)^{d-2} \left( \sum_{n,m=-\infty}^{\infty} (\xi^2 n^2 + m^2)^{\frac{d-2}{2}} - \sum_{n,m=-\infty}^{\infty} \left( \xi^2 \left(n + \frac{1}{2}\right)^2 + \left(m + \frac{1}{2}\right)^2 \right)^{\frac{d-2}{2}} \right). \tag{26}$$

After taking care of the infrared singularity in the bosonic sector [2], one obtains finally

$$F = -\frac{\Gamma\left(\frac{d}{2}\right) \xi^d \pi^{-\frac{d}{2}}}{L^{d-1}} \left( Z_2 \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) - Z_2 \begin{vmatrix} 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix} \left(\frac{d}{2}, 1, \xi^2\right) \right), \tag{27}$$

and using the definition of (17) and the Epstein-zeta properties we obtain

$$f(\xi) = \xi^d f\left(\frac{1}{\xi}\right). \tag{28}$$

One can easily check that the high temperature free energy is equal to the zero-temperature Casimir effect, which is expressed from (28).

## 3. Conclusions

Studies of the dualities that physical systems have are of great importance. In this paper, a kind of duality was examined, expressed in terms of the temperature inversion symmetry of thermodynamic quantities for a supersymmetric ensemble ($N = 1$, $d = 4$ Wess–Zumino with $S^1 \times R^2$ spatial topology, at finite temperature). This symmetry connects the free energy at high temperatures (Boltzmann law), with the zero-temperature Casimir energy. Two cases were studied in this work. (a) The first case, with the chiral superfield of the Wess–Zumino model, consists of periodic fermions and antiperiodic bosons in the compact dimension. In this case, we found that the function $f(\xi)$, defined in (17), is antisymmetric under the transformation $\xi \rightarrow \frac{1}{\xi}$, i.e. $f(\xi) = -\xi^d f\left(\frac{1}{\xi}\right)$. Thus the high-temperature free energy of the boson fermion ensemble is equal to the minus zero-temperature Casimir energy. (b) The second case in which periodic bosons and antiperiodic fermions in the compact dimension were used. In this case, the function $f(\xi)$ was found symmetric under the transformation $\xi \rightarrow \frac{1}{\xi}$, i.e. $f(\xi) = \xi^d f\left(\frac{1}{\xi}\right)$. Consequently, the high-temperature free energy is equal to the zero-temperature Casimir energy. The function $f(\xi)$ is related to the free energy of the system under consideration and the results show how boundary conditions can affect the quantum structure of the system, in terms of $f(\xi)$. Finally, one can easily observe that if $\xi = 1$, then the free energy of the system, composed of periodic fermions and antiperiodic bosons, becomes equal to zero. The $\xi = 1$ limit corresponds to the case $L = \frac{1}{T}$. If interactions were added, then, maybe, this limit could connect phase transitions at some critical temperature, with a dual critical length. If the free energy remains zero at this limit, the question is what the physics of a system with zero free energy at a temperature $T \neq 0$ is (one system having zero free energy is the d5 Ads black hole with Ricci flat horizons). The zero of the free energy occurs for the Hawking temperature $T_{\mathrm{BH}} = \frac{1}{l n_{\mathrm{BH}}}$ ($l$ the radius of the asymptotic Ads space), and determines the critical point of the transition, Ads black hole to Ads soliton [13]).

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